

## Topic 5 Part 2 [360 marks]

1a. [1 mark]

### Markscheme

0 *AI*

[1 mark]

### Examiners report

A range of answers were seen to part a), though many more could have gained the mark had they taken time to understand the shape of the function. Part b) was done well, as was part c). In c), a number of candidates integrated by parts, but found the incorrect expression

$$-xe^{-x} + e^{-x}.$$

1b. [3 marks]

### Markscheme

$$\int_0^1 f(x)dx = 1 \quad (MI)$$

$$\Rightarrow a = \frac{1}{\int_0^1 e^{-x} dx}$$

$$\Rightarrow a = \frac{1}{[-e^{-x}]_0^1}$$

$$\Rightarrow a = \frac{e}{e-1} \text{ (or equivalent)} \quad AI$$

**Note:** Award first *AI* for correct integration of

$$\int e^{-x} dx.$$

This *AI* is independent of previous *M* mark.

[3 marks]

### Examiners report

A range of answers were seen to part a), though many more could have gained the mark had they taken time to understand the shape of the function. Part b) was done well, as was part c). In c), a number of candidates integrated by parts, but found the incorrect expression

$$-xe^{-x} + e^{-x}.$$

1c. [4 marks]

### Markscheme

$$E(X) = \int_0^1 xf(x)dx \left( = a \int_0^1 xe^{-x} dx \right) \quad MI$$

attempt to integrate by parts *MI*

$$= a[-xe^{-x} - e^{-x}]_0^1 \quad (AI)$$

$$= a\left(\frac{e-2}{e}\right)$$

$$= \frac{e-2}{e-1} \text{ (or equivalent)} \quad AI$$

[4 marks]

### Examiners report

A range of answers were seen to part a), though many more could have gained the mark had they taken time to understand the shape of the function. Part b) was done well, as was part c). In c), a number of candidates integrated by parts, but found the incorrect expression

$$-xe^{-x} + e^{-x}.$$

2.

[5 marks]

## Markscheme

$$\int_0^{\frac{\pi}{2}} x \sin x \, dx \quad \text{M1}$$

$$= [-x \cos x]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x \, dx \quad \text{M1(A1)}$$

**Note:** Condone the absence of limits or wrong limits to this point.

$$= [-x \cos x + \sin x]_0^{\frac{\pi}{2}} \quad \text{A1}$$

$$= 1 \quad \text{A1}$$

[5 marks]

## Examiners report

It was pleasing to note how many candidates recognised the expression that needed to be integrated and successfully used integration by parts to reach the correct answer.

## Markscheme

**Note:** Be aware that an unjustified assumption of independence will also lead to  $P(B) = 0.25$ , but is an invalid method.

### METHOD 1

$$P(A'|B') = 1 - P(A|B') = 1 - 0.6 = 0.4 \quad \text{MIAI}$$

$$P(A'|B') = \frac{P(A' \cap B')}{P(B')}$$

$$P(A' \cap B') = P((A \cup B)') = 1 - 0.7 = 0.3 \quad \text{AI}$$

$$0.4 = \frac{0.3}{P(B')} \Rightarrow P(B') = 0.75 \quad \text{(MI)AI}$$

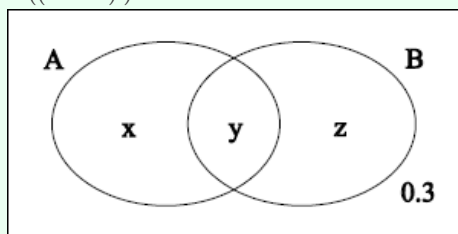
$$P(B) = 0.25 \quad \text{AI}$$

(this method can be illustrated using a tree diagram)

[6 marks]

### METHOD 2

$$P((A \cup B)') = 1 - 0.7 = 0.3 \quad \text{AI}$$



$$P(A|B') = \frac{x}{x+0.3} = 0.6 \quad \text{MIAI}$$

$$x = 0.6x + 0.18$$

$$0.4x = 0.18$$

$$x = 0.45 \quad \text{AI}$$

$$P(A \cup B) = x + y + z$$

$$P(B) = y + z = 0.7 - 0.45 \quad \text{(MI)}$$

$$= 0.25 \quad \text{AI}$$

[6 marks]

### METHOD 3

$$\frac{P(A \cap B')}{P(B')} = 0.6 \quad (\text{or } P(A \cap B') = 0.6P(B')) \quad \text{MI}$$

$$P(A \cap B') = P(A \cup B) - P(B) \quad \text{MIAI}$$

$$P(B') = 1 - P(B)$$

$$0.7 - P(B) = 0.6 - 0.6P(B) \quad \text{MI(AI)}$$

$$0.1 = 0.4P(B)$$

$$P(B) = \frac{1}{4} \quad \text{AI}$$

[6 marks]

## Examiners report

There is a great variety of ways to approach this question and there were plenty of very good solutions produced, all of which required an insight into the structure of conditional probability. A few candidates unfortunately assumed independence and so did not score well.

4a.

[3 marks]

## Markscheme

$$B\left(6, \frac{2}{3}\right) \quad (M1)$$

$$p(4) = \binom{6}{4} \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 \quad A1$$

$$\binom{6}{4} = 15 \quad A1$$

$$= 15 \times \frac{2^4}{3^6} = \frac{80}{243} \quad AG$$

[3 marks]

## Examiners report

This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

(a) Candidates need to be aware how to work out binomial coefficients without a calculator

4b.

[4 marks]

## Markscheme

(i) 2 outcomes for each of the 6 games or

$$2^6 = 64 \quad R1$$

(ii)

$$(1+x)^6 = \binom{6}{0} + \binom{6}{1}x + \binom{6}{2}x^2 + \binom{6}{3}x^3 + \binom{6}{4}x^4 + \binom{6}{5}x^5 + \binom{6}{6}x^6 \quad A1$$

**Note:** Accept

${}^nC_r$  notation or

$$1 + 6x + 15x^2 + 20x^3 + 15x^4 + 6x^5 + x^6$$

setting  $x = 1$  in both sides of the expression  $R1$

**Note:** Do not award **R1** if the right hand side is not in the correct form.

$$64 = \binom{6}{0} + \binom{6}{1} + \binom{6}{2} + \binom{6}{3} + \binom{6}{4} + \binom{6}{5} + \binom{6}{6} \quad AG$$

(iii) the total number of outcomes = number of ways Alfred can win no games, plus the number of ways he can win one game *etc.*

**R1**

[4 marks]

## Examiners report

This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

(b) (ii) A surprising number of candidates chose to work out the values of all the binomial coefficients (or use Pascal's triangle) to make a total of 64 rather than simply putting 1 into the left hand side of the expression.

## Markscheme

(i) Let

$P(x, y)$  be the probability that Alfred wins  $x$  games on the first day and  $y$  on the second.

$$P(4, 2) = \binom{6}{4} \times \left(\frac{2}{3}\right)^4 \times \left(\frac{1}{3}\right)^2 \times \binom{6}{2} \times \left(\frac{2}{3}\right)^2 \times \left(\frac{1}{3}\right)^4 \quad \text{MIAI}$$

$$\binom{6}{2}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6 \text{ or}$$

$$\binom{6}{4}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6 \quad \text{AI}$$

$r = 2$  or  $4$ ,  $s = t = 6$

(ii)  $P(\text{Total} = 6) =$

$$P(0, 6) + P(1, 5) + P(2, 4) + P(3, 3) + P(4, 2) + P(5, 1) + P(6, 0) \quad (\text{MI})$$

$$= \binom{6}{0}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6 + \binom{6}{1}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6 + \dots + \binom{6}{6}^2 \left(\frac{2}{3}\right)^6 \left(\frac{1}{3}\right)^6 \quad \text{A2}$$

$$= \frac{2^6}{3^{12}} \left( \binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2 \right)$$

**Note:** Accept any valid sum of 7 probabilities.

(iii) use of

$$\binom{6}{i} = \binom{6}{6-i} \quad (\text{MI})$$

(can be used either here or in (c)(ii))

$P(\text{wins 6 out of 12})$

$$= \binom{12}{6} \times \left(\frac{2}{3}\right)^6 \times \left(\frac{1}{3}\right)^6 = \frac{2^6}{3^{12}} \binom{12}{6} \quad \text{AI}$$

$$= \frac{2^6}{3^{12}} \left( \binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2 \right) = \frac{2^6}{3^{12}} \binom{12}{6} \quad \text{AI}$$

therefore

$$\binom{6}{0}^2 + \binom{6}{1}^2 + \binom{6}{2}^2 + \binom{6}{3}^2 + \binom{6}{4}^2 + \binom{6}{5}^2 + \binom{6}{6}^2 = \binom{12}{6} \quad \text{AG}$$

[9 marks]

## Examiners report

This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

4d.

[6 marks]

## Markscheme

(i)

$$E(A) = \sum_{r=0}^n r \binom{n}{r} \left(\frac{2}{3}\right)^r \left(\frac{1}{3}\right)^{n-r} = \sum_{r=0}^n r \binom{n}{r} \frac{2^r}{3^n}$$

$$(a = 2, b = 3) \quad \text{MIAI}$$

**Note:** *M0A0* for  $a = 2, b = 3$  without any method.

(ii)

$$n(1+x)^{n-1} = \sum_{r=1}^n \binom{n}{r} r x^{r-1} \quad \text{AIAI}$$

(sigma notation not necessary)

(if sigma notation used also allow lower limit to be  $r = 0$ )

$$\text{let } x = 2 \quad \text{MI}$$

$$n3^{n-1} = \sum_{r=1}^n \binom{n}{r} r 2^{r-1}$$

multiply by 2 and divide by

$$3^n \quad \text{(MI)}$$

$$\frac{2n}{3} = \sum_{r=1}^n \binom{n}{r} r \frac{2^r}{3^n} \left( = \sum_{r=0}^n \binom{n}{r} \frac{2^r}{3^n} \right) \quad \text{AG}$$

[6 marks]

## Examiners report

This question linked the binomial distribution with binomial expansion and coefficients and was generally well done.

(d) This was poorly done. Candidates were not able to manipulate expressions given using sigma notation.

5.

[4 marks]

## Markscheme

$$\frac{5 \times 6 + 6k + 7 \times 3 + 8 \times 1 + 9 \times 2 + 10 \times 1}{13 + k} = 6.5 \text{ (or equivalent)} \quad \text{(MI)(AI)(AI)}$$

**Note:** Award *(MI)(AI)* for correct numerator, and *(AI)* for correct denominator.

$$0.5k = 2.5 \Rightarrow k = 5 \quad \text{AI}$$

[4 marks]

## Examiners report

The question was well done generally as one would expect.

6a.

[2 marks]

## Markscheme

Let  $X$  represent the length of time a journey takes on a particular day.

$$P(X > 15) = 0.0912112819 \dots = 0.0912 \quad \text{(MI)AI}$$

## Examiners report

There were many good answers to this question. Some students lost accuracy marks by early rounding. Some students struggled with the Binomial distribution.

6b.

[3 marks]

## Markscheme

Use of correct Binomial distribution (MI)

$$N \sim B(5, 0.091\dots)$$

$$1 - 0.0912112819\dots = 0.9087887181\dots$$

$$1 - (0.9087887181\dots)^5 = 0.380109935\dots = 0.380 \quad (MI)AI$$

**Note:** Allow answers to be given as percentages.

[5 marks]

## Examiners report

There were many good answers to this question. Some students lost accuracy marks by early rounding. Some students struggled with the Binomial distribution.

7a.

[3 marks]

## Markscheme

$$X \sim \text{Po}(0.25T) \quad (AI)$$

Attempt to solve

$$P(X \leq 3) = 0.6 \quad (MI)$$

$$T = 12.8453\dots = 13 \text{ (minutes)} \quad AI$$

**Note:** Award AIMIA0 if  $T$  found correctly but not stated to the nearest minute.

[3 marks]

## Examiners report

There were some good answers to part (a), although poor calculator use frequently let down the candidates.

7b.

[4 marks]

$$X_1$$

$$X_2$$

$$X_1$$

$$X_2$$

$$= P(X_1 \leq 3) \times P(X_2 \leq 3) + P(X_1 = 4) \times P(X_2 \leq 2)$$

$$+ P(X_1 = 5) \times P(X_2 \leq 1) + P(X_1 = 6) \times P(X_2 = 0)$$

$$= 0.573922\dots + 0.072654\dots + 0.019192\dots + 0.002285\dots$$

$$= 0.668 \text{ (053\dots)}$$

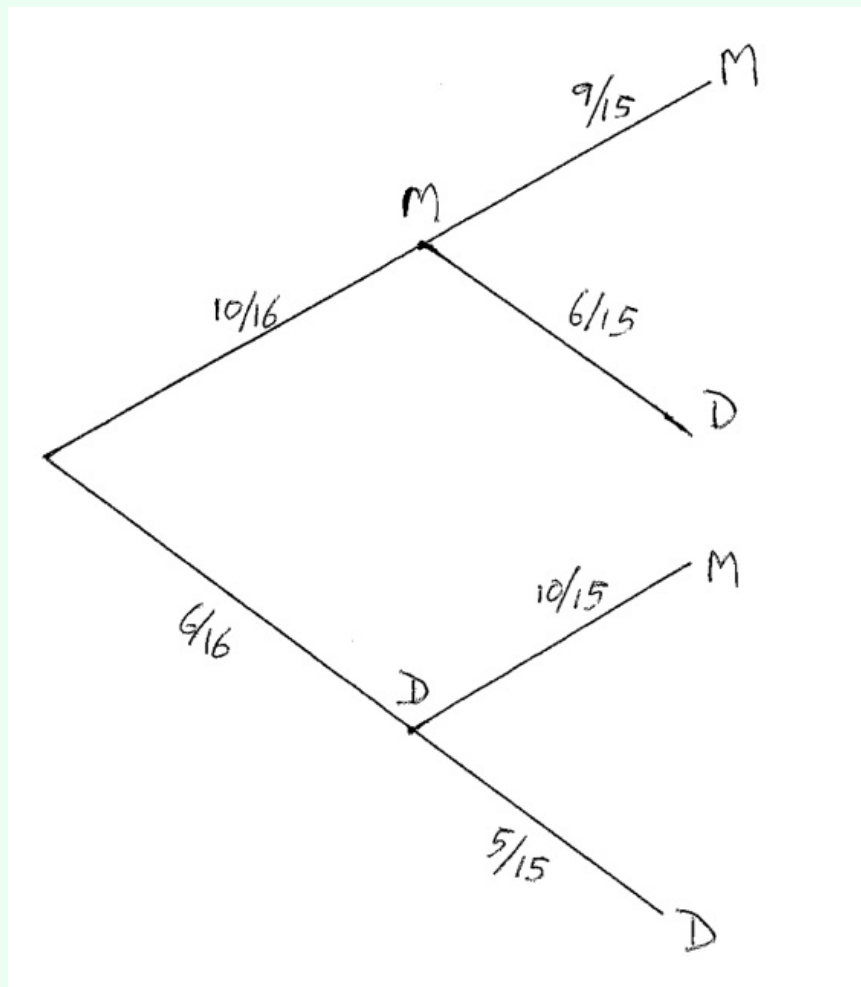
## Examiners report

Very few candidates were able to access part (b).

8a.

[3 marks]

### Markscheme



*AIAIAI*

[3 marks]

**Note:** Award *A1* for the initial level probabilities, *A1* for each of the second level branch probabilities.

## Examiners report

Generally well done. A few candidates didn't take account of the fact that Caz ate the chocolate, so didn't replace it. A few candidates made arithmetic errors in calculating the probability.

8b.

[2 marks]

### Markscheme

$$\frac{10}{16} \times \frac{9}{15} + \frac{6}{16} \times \frac{5}{15} \quad (M1)$$

$$= \frac{120}{240} \left( = \frac{1}{2} \right) \quad A1$$

[2 marks]



## Examiners report

Generally well done. A few candidates didn't take account of the fact that Caz ate the chocolate, so didn't replace it. A few candidates made arithmetic errors in calculating the probability.

9a.

[2 marks]

### Markscheme

$$X \sim N(13.5, 9.5)$$

$$13.5 - \sqrt{9.5} < X < 13.5 + \sqrt{9.5} \quad (M1)$$

$$10.4 < X < 16.6 \quad A1$$

**Note:** Accept 6.16.

[2 marks]

## Examiners report

A large proportion of candidates experienced difficulties with this question. In parts (a) and (b), the most common error was to use  $\sigma = 9.5$ . In part (a), a large number of candidates used their range of values to then unnecessarily find the corresponding probability of that time interval occurring. In part (b), a large number of candidates used an unrealistic lower bound (a large negative value) for time.

9b.

[3 marks]

### Markscheme

$$P(X < 10) = 0.12807 \dots \quad (M1)(A1)$$

estimate is 1281 (correct to the nearest whole number).  $A1$

**Note:** Accept 1280.

[3 marks]

## Examiners report

A large proportion of candidates experienced difficulties with this question. In parts (a) and (b), the most common error was to use  $\sigma = 9.5$ . In part (a), a large number of candidates used their range of values to then unnecessarily find the corresponding probability of that time interval occurring. In part (b), a large number of candidates used an unrealistic lower bound (a large negative value) for time.

10a.

[3 marks]

## Markscheme

$$\int_0^{0.5} ax^2 dx + \int_{0.5}^1 0.5a(1-x) dx = 1 \quad \text{MIAI}$$

$$\frac{5a}{48} \text{ (or equivalent) or}$$

$$a \times 0.104\dots = 1 \quad \text{AI}$$

**Note:** Award **MI** for considering two definite integrals.

Award **AI** for equating to 1.

Award **AI** for a correct equation.

The **AIAI** can be awarded in any order.

$$a = 9.6 \quad \text{AG}$$

[3 marks]

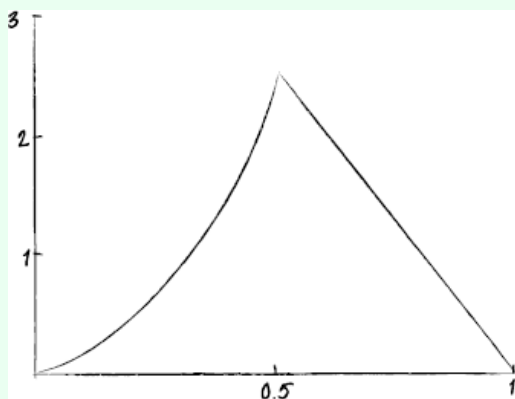
## Examiners report

Part (a) was generally well done. Common errors usually involved not recognizing that the sum of the two integrals was equal to one, premature rounding or not showing full working to conclusively show that  $a = 9.6$ .

10b.

[2 marks]

## Markscheme



correct shape for

$$0 \leq x \leq 0.5 \text{ and}$$

$$f(0.5) \approx 2.4 \quad \text{AI}$$

correct shape for

$$0.5 \leq x \leq 1 \text{ and}$$

$$f(1) = 0 \quad \text{AI}$$

[2 marks]

## Examiners report

Part (b) was not well done with many graphs poorly labelled and offering no reference to domain and range.

10c. [2 marks]

## Markscheme

attempting to find

$$P(X < 0.6) \quad (MI)$$

direct GDC use or *eg*

$$P(0 \leq X \leq 0.5) + P(0.5 \leq X \leq 0.6) \text{ or}$$

$$1 - P(0.6 \leq X \leq 1)$$

$$P(X < 0.6) = 0.616 \left( = \frac{77}{125} \right) \quad AI$$

[2 marks]

## Examiners report

Part (c) was reasonably well done. The most common error involved calculating an incorrect probability from an incorrect definite integral.

11a. [2 marks]

## Markscheme

$$X \sim \text{Po}(1.2)$$

$$P(X = 3) \times P(X = 0) \quad (MI)$$

$$= 0.0867 \dots \times 0.3011 \dots$$

$$= 0.0261 \quad AI$$

[2 marks]

## Examiners report

Part (a) was generally well done although a number of candidates added the two probabilities rather than multiplying the two probabilities. A number of candidates specified the required probability correct to two significant figures only.

11b. [5 marks]

## Markscheme

Three requests over two days can occur as (3, 0), (0, 3), (2, 1) or (1, 2). *RI*

using conditional probability, for example

$$\frac{P(3, 0)}{P(3 \text{ requests}, m=2.4)} = 0.125 \text{ or } \frac{P(2, 1)}{P(3 \text{ requests}, m=2.4)} = 0.375 \quad MIAI$$

expected income is

$$2 \times 0.125 \times \text{US\$}120 + 2 \times 0.375 \times \text{US\$}180 \quad MI$$

**Note:** Award *MI* for attempting to find the expected income including both (3, 0) and (2, 1) cases.

$$= \text{US\$}30 + \text{US\$}135$$

$$= \text{US\$}165 \quad AI$$

[5 marks]

## Examiners report

Part (b) challenged most candidates with only a few candidates able to correctly employ a conditional probability argument.

12a. [4 marks]

### Markscheme

(i)

$$\sum_{k=1}^n (2k-1) \text{ (or equivalent)} \quad \text{AI}$$

**Note:** Award *A0* for

$$\sum_{n=1}^n (2n-1) \text{ or equivalent.}$$

(ii) **EITHER**

$$2 \times \frac{n(n+1)}{2} - n \quad \text{MIAI}$$

**OR**

$$\frac{n}{2}(2 + (n-1)2) \text{ (using } S_n = \frac{n}{2}(2u_1 + (n-1)d)) \quad \text{MIAI}$$

**OR**

$$\frac{n}{2}(1 + 2n - 1) \text{ (using } S_n = \frac{n}{2}(u_1 + u_n)) \quad \text{MIAI}$$

**THEN**

$$= n^2 \quad \text{AG}$$

(iii)

$$47^2 - 14^2 = 2013 \quad \text{AI}$$

[4 marks]

## Examiners report

In part (a) (i), a large number of candidates were unable to correctly use sigma notation to express the sum of the first  $n$  positive odd integers. Common errors included summing

$2n - 1$  from 1 to  $n$  and specifying sums with incorrect limits. Parts (a) (ii) and (iii) were generally well done.

12b.

[7 marks]

## Markscheme

(i) **EITHER**a pentagon and five diagonals **AI****OR**five diagonals (circle optional) **AI**(ii) Each point joins to  $n - 3$  other points. **AI**

a correct argument for

$$n(n - 3) \quad \textbf{RI}$$

a correct argument for

$$\frac{n(n-3)}{2} \quad \textbf{RI}$$

(iii) attempting to solve

$$\frac{1}{2}n(n - 3) > 1\,000\,000 \text{ for } n. \quad \textbf{(MI)}$$

$$n > 1415.7 \quad \textbf{(AI)}$$

$$n = 1416 \quad \textbf{AI}$$

**[7 marks]**

## Examiners report

Parts (b) (i) and (iii) were generally well done. In part (b) (iii), many candidates unnecessarily simplified their quadratic when direct GDC use could have been employed. A few candidates gave

$n > 1416$  as their final answer. While some candidates displayed sound reasoning in part (b) (ii), many candidates unfortunately adopted a ‘proof by example’ approach.

12c.

[8 marks]

## Markscheme

(i)  $np = 4$  and  $npq = 3$  **(AI)**attempting to solve for  $n$  and  $p$  **(MI)**

$$n = 16 \text{ and}$$

$$p = \frac{1}{4} \quad \textbf{AI}$$

(ii)

$$X \sim B(16, 0.25) \quad \textbf{(AI)}$$

$$P(X = 1) = 0.0534538... (= \binom{16}{1} (0.25)(0.75)^{15}) \quad \textbf{(AI)}$$

$$P(X = 3) = 0.207876... (= \binom{16}{3} (0.25)^3 (0.75)^{13}) \quad \textbf{(AI)}$$

$$P(X = 1) + P(X = 3) \quad \textbf{(MI)}$$

$$= 0.261 \quad \textbf{AI}$$

**[8 marks]**

## Examiners report

Part (c) was generally well done. In part (c) (ii), some candidates multiplied the two probabilities rather than adding the two probabilities.

13a.

[5 marks]

### Markscheme

(i)

$$P(X = 6) = 0.122 \quad (MI)AI$$

(ii)

$$P(X = 6 | 5 \leq X \leq 8) = \frac{P(X=6)}{P(5 \leq X \leq 8)} = \frac{0.122 \dots}{0.592 \dots - 0.0996 \dots} \quad (MI)(AI)$$
$$= 0.248 \quad AI$$

[5 marks]

## Examiners report

[N/A]

13b.

[3 marks]

### Markscheme

(i)

$$E(\bar{X}) = 8 \quad AI$$

$$\text{Var}(\bar{X}) = \frac{8}{n} \quad AI$$

(ii)

$$E(\bar{X}) \neq \text{Var}(\bar{X})$$

$$(\text{for } n > 1) \quad RI$$

**Note:** Only award the **RI** if the two expressions in (b)(i) are different.

[3 marks]

## Examiners report

[N/A]

## Markscheme

(i) **EITHER**

$$\bar{X} \sim N(8, 0.2) \quad (MI)AI$$

**Note:** *MI* for normality, *AI* for parameters.

$$P(7.1 < \bar{X} < 8.5) = 0.846 \quad AI$$

**OR**

The expression is equivalent to

$$P(283 \leq \sum X \leq 339) \text{ where}$$

$$\sum X \text{ is}$$

$$Po(320) \quad MIAI$$

$$= 0.840 \quad AI$$

**Note:** Accept 284, 340 instead of 283, 339

Accept any answer that rounds correctly to 0.84 or 0.85.

(ii) **EITHER**

$$k = 1.96 \frac{\sigma}{\sqrt{n}} \text{ or}$$

$$1.96 \text{ std}(\bar{X}) \quad (MI)(AI)$$

$$k = 0.877 \text{ or}$$

$$1.96\sqrt{0.2} \quad AI$$

**OR**

The expression is equivalent to

$$P(320 - 40k \leq \sum X \leq 320 + 40k) = 0.95 \quad (MI)$$

$$k = 0.875 \quad A2$$

**Note:** Accept any answer that rounds to 0.87 or 0.88.

Award *MIA0* if modulus sign ignored and answer obtained rounds to 0.74 or 0.75

[6 marks]

## Examiners report

[N/A]

## Markscheme

(a)

$$a \int_0^{\frac{\pi}{2}} x \cos x dx = 1 \quad (M1)$$

integrating by parts:

$$u = x$$

$$v' = \cos x \quad M1$$

$$u' = 1$$

$$v = \sin x$$

$$\int x \cos x dx = x \sin x + \cos x \quad A1$$

$$[x \sin x + \cos x]_0^{\frac{\pi}{2}} = \frac{\pi}{2} - 1 \quad A1$$

$$a = \frac{1}{\frac{\pi}{2} - 1} \quad A1$$

$$= \frac{2}{\pi - 2} \quad AG$$

[5 marks]

(b)

$$P\left(X < \frac{\pi}{4}\right) = \frac{2}{\pi - 2} \int_0^{\frac{\pi}{4}} x \cos x dx = 0.460 \quad (M1)A1$$

**Note:** Accept

$$\frac{2}{\pi - 2} \left( = \frac{\pi\sqrt{2}}{8} + \frac{\sqrt{2}}{2} - 1 \right) \text{ or equivalent}$$

[2 marks]

(c) (i)

$$\text{mode} = 0.860 \quad A1$$

(x-value of a maximum on the graph over the given domain)

(ii)

$$\frac{2}{\pi - 2} \int_0^m x \cos x dx = 0.5 \quad (M1)$$

$$\int_0^m x \cos x dx = \frac{\pi - 2}{4}$$

$$m \sin m + \cos m - 1 = \frac{\pi - 2}{4} \quad (M1)$$

$$\text{median} = 0.826 \quad A1$$

**Note:** Do not accept answers containing additional solutions.

[4 marks]

(d)

$$P\left(X < \frac{\pi}{8} | X < \frac{\pi}{4}\right) = \frac{P\left(X < \frac{\pi}{8}\right)}{P\left(X < \frac{\pi}{4}\right)} \quad M1$$

$$= \frac{0.129912}{0.459826}$$

$$= 0.283 \quad A1$$

[2 marks]

Total [13 marks]

## Examiners report

[N/A]



15.

[6 marks]

## Markscheme

(a)

$$\frac{1}{6} + \frac{1}{2} + \frac{3}{10} + a = 1 \Rightarrow a = \frac{1}{30} \quad \text{AI}$$

(b)

$$E(X) = \frac{1}{2} + 2 \times \frac{3}{10} + 3 \times \frac{1}{30} \quad \text{MI}$$

$$= \frac{6}{5} \quad \text{AI}$$

**Note:** Do not award *FT* marks if  $a$  is outside  $[0, 1]$ .

[2 marks]

(c)

$$E(X^2) = \frac{1}{2} + 2^2 \times \frac{3}{10} + 3^2 \times \frac{1}{30} = 2 \quad (\text{AI})$$

attempt to apply

$$\text{Var}(X) = E(X^2) - (E(X))^2 \quad \text{MI}$$

$$\left( = 2 - \frac{36}{25} \right) = \frac{14}{25} \quad \text{AI}$$

[3 marks]

Total [6 marks]

## Examiners report

This was very well answered and many fully correct solutions were seen. A small number of candidates made arithmetic mistakes in part a) and thus lost one or two accuracy marks. A few also seemed unaware of the formula

$\text{Var}(X) = E(X^2) - E(X)^2$  and resorted to seeking an alternative, sometimes even attempting to apply a clearly incorrect

$$\text{Var}(X) = \sum (x_i - \mu)^2.$$

16.

[6 marks]

$$P\left(Z < \frac{780 - \mu}{\sigma}\right) = 0.92$$

$$P\left(Z < \frac{755 - \mu}{\sigma}\right) = 0.12$$

$$\Rightarrow \frac{780 - \mu}{\sigma} = 1.405 \dots$$

$$\frac{755 - \mu}{\sigma} = -1.174 \dots$$

$$\mu = 766.385$$

$$\sigma = 9.6897$$

$$\mu = 12 \text{ hrs } 46 \text{ mins } (= 766 \text{ mins})$$

$$\sigma = 10 \text{ mins}$$

## Examiners report

Generally well done. Most candidates made correct use of the symmetry of the normal curve and the inverse normal to set up a correct pair of equations involving

$\mu$  and

$\sigma$ . A few candidates expressed equations containing the GDC command term invNorm.

A few candidates did not express their answers correct to the nearest minute and a few candidates performed erroneous conversions from hours to minutes.

17a. [3 marks]

## Markscheme

$$P(F) = \left(\frac{1}{7} \times \frac{7}{9}\right) + \left(\frac{6}{7} \times \frac{4}{9}\right) \quad (MI)(AI)$$

**Note:** Award **MI** for the sum of two products.

$$= \frac{31}{63} (= 0.4920\dots) \quad AI$$

[3 marks]

## Examiners report

Both parts were very well done. In part (a), most candidates successfully used a tree diagram.

17b. [2 marks]

## Markscheme

Use of

$$P(S|F) = \frac{P(S \cap F)}{P(F)} \text{ to obtain}$$

$$P(S|F) = \frac{\frac{1}{7} \times \frac{7}{9}}{\frac{31}{63}}. \quad MI$$

**Note:** Award **MI** only if the numerator results from the product of two probabilities.

$$= \frac{7}{31} (= 0.2258\dots) \quad AI$$

[2 marks]

## Examiners report

Both parts were very well done. In part (b), most candidates correctly used conditional probability considerations.

18a.

[9 marks]

## Markscheme

(i)

$$X \sim \text{Po}(0.6)$$

$$P(X = 0) = 0.549 \quad (= e^{-0.6}) \quad \text{AI}$$

(ii)

$$\begin{aligned} P(X \geq 3) &= 1 - P(X \leq 2) \quad (\text{MI})(\text{AI}) \\ &= 1 - \left( e^{-0.6} + e^{-0.6} \times 0.6 + e^{-0.6} \times \frac{0.6^2}{2} \right) \\ &= 0.0231 \quad \text{AI} \end{aligned}$$

(iii)

$$Y \sim \text{Po}(2.4) \quad (\text{MI})$$

$$P(Y \leq 5) = 0.964 \quad \text{AI}$$

(iv)

$$Z \sim \text{B}(12, 0.451 \dots) \quad (\text{MI})(\text{AI})$$

**Note:** Award **MI** for recognising binomial and **AI** for using correct parameters.

$$P(Z = 4) = 0.169 \quad \text{AI}$$

[9 marks]

## Examiners report

Parts (a) and (b) were generally well done by a large proportion of candidates. In part (a) (ii), some candidates used an incorrect inequality (e.g.

$P(X \geq 3) = 1 - P(X \leq 3)$ ) while in (a) (iii) some candidates did not use

$\mu = 2.4$ . In part (a) (iv), a number of candidates either did not realise that they needed to consider a binomial random variable or did so using incorrect parameters.

18b.

[9 marks]

## Markscheme

(i)

$$k \int_1^3 \ln x \, dx = 1 \quad (\text{MI})$$

$$(k \times 1.2958 \dots = 1)$$

$$k = 0.771702 \quad \text{AI}$$

(ii)

$$E(X) = \int_1^3 kx \ln x \, dx \quad (\text{AI})$$

attempting to evaluate their integral  $(\text{MI})$

$$= 2.27 \quad \text{AI}$$

(iii)

$$x = 3 \quad \text{AI}$$

(iv)

$$\int_1^m k \ln x \, dx = 0.5 \quad (\text{MI})$$

$$k[x \ln x - x]_1^m = 0.5$$

attempting to solve for  $m$   $(\text{MI})$

$$m = 2.34 \quad \text{AI}$$

[9 marks]

## Examiners report

Parts (a) and (b) were generally well done by a large proportion of candidates.

In (b) (i), some candidates gave their value of  $k$  correct to three significant figures rather than correct to six decimal places. In parts (b) (i), (ii) and (iv), a large number of candidates unnecessarily used integration by parts. In part (b) (iii), a number of candidates thought the mode of  $X$  was

$f(3)$  rather than

$x = 3$ . In part (b) (iv), a number of candidates did not consider the domain of  $f$  when attempting to find the median or checking their solution.

19a. [4 marks]

### Markscheme

(i)

$$\bar{v} = \frac{1}{1000}(55 \times 5 + 65 \times 13 + \dots + 145 \times 31) \quad \text{AIMI}$$

**Note:** *AI* for mid-points, *MI* for use of the formula.

$$= \frac{113\,210}{1000} = 113.21 \quad \text{AG}$$

(ii)

$$s^2 = \frac{(55-113.21)^2 \times 5 + (65-113.21)^2 \times 13 + \dots + (145-113.21)^2 \times 31}{999} \quad \text{(MI)}$$

$$= \frac{362\,295.9}{999} = 362.6585\dots = 363 \quad \text{AI}$$

**Note:** Award *AI* if answer rounds to 362 or 363.

**Note:** Condone division by 1000.

[4 marks]

## Examiners report

In (a)(i), the candidates were required to show that the estimate of the mean is 113.21 so that those who stated simply ‘Using my GDC, mean = 113.21’ were given no credit. Candidates were expected to indicate that the interval midpoints were used and to show the appropriate formula. In (a)(ii), division by either 999 or 1000 was accepted, partly because of the large sample size and partly because the question did not ask for an unbiased estimate of variance.

19b. [2 marks]

### Markscheme

$$\bar{v} \pm \frac{t_{0.025} \times s}{\sqrt{n}} \quad \text{(MI)}$$

hence the confidence interval

$$I = [112.028, 114.392] \quad \text{AI}$$

**Note:** Accept answers which round to 112 and 114.

**Note:** Condone the use of

$z_{0.025}$  for

$t_{0.025}$  and

$\sigma$  for

$s$ .

[2 marks]

## Examiners report

19c. [2 marks]

### Markscheme

less confidence implies narrower interval **R2**

**Note:** Accept equivalent statements or arguments having a meaningful diagram and/or relevant percentiles.  
hence the confidence interval

$I$  at the 95% level contains the confidence interval

$J$  at the 90% level **AG**

[2 marks]

### Examiners report

Solutions to (c) were often badly written, often quite difficult to understand exactly what was being stated.

20. [4 marks]

### Markscheme

$$k \int_1^2 2^{\frac{1}{x}} dx = 1 \Rightarrow k = \frac{1}{\int_1^2 2^{\frac{1}{x}} dx} (= 0.61556...) \quad (MI)(AI)$$

$$E(X) = k \int_1^2 x 2^{\frac{1}{x}} dx = 2.39...k \text{ or } 1.47 \quad MIAI$$

**Note:** Condone missing  $dx$  in any part of the question.

[4 marks]

### Examiners report

This question was well attempted by most candidates. However many were not alert for the necessity of using GDC to calculate the definite integrals and wasted time trying to obtain these values using standard calculus methods without success.

21a. [2 marks]

### Markscheme

$$\binom{10}{6} = 210 \quad (MI)AI$$

[2 marks]

### Examiners report

Most candidates answered this question well although in some cases candidates were not able to distinguish the use of permutations from combinations. Almost all candidates scored the two marks of part (c), but many of these were follow through marks.

21b. [3 marks]

### Markscheme

$$2 \times \binom{8}{5} = 112 \quad (MI)AIAI$$

**Note:** Accept  
 $210 - 28 - 70 = 112$

[3 marks]

### Examiners report

Most candidates answered this question well although in some cases candidates were not able to distinguish the use of permutations from combinations. Almost all candidates scored the two marks of part (c), but many of these were follow through marks.

21c. [2 marks]

### Markscheme

$$\frac{112}{210} \left( = \frac{8}{15} = 0.533 \right) \quad (MI)AI$$

[2 marks]

### Examiners report

Most candidates answered this question well although in some cases candidates were not able to distinguish the use of permutations from combinations. Almost all candidates scored the two marks of part (c), but many of these were follow through marks.

22a. [1 mark]

### Markscheme

50 *AI*

[1 mark]

### Examiners report

Very few candidates were successful in answering this question. In many cases it was clear that candidates were not familiar with box-and-whisker plots at all; in other cases the explanations given revealed various misconceptions.

22b. [4 marks]

### Markscheme

Lower quartile is 4 so at least 26 obtained a 4 *RI*  
 Lower bound is 26 *AI*  
 Minimum is 2 but the rest could be 4 *RI*  
 So upper bound is 49 *AI*  
**Note:** Do not allow follow through for *A* marks.

**Note:** If answers are incorrect award *R0A0*; if argument is correct but no clear lower/upper bound is stated award *RIA0*; award *R0AI* for correct answer without explanation or incorrect explanation.

[4 marks]

### Examiners report

Very few candidates were successful in answering this question. In many cases it was clear that candidates were not familiar with box-and-whisker plots at all; in other cases the explanations given revealed various misconceptions.

23.

[5 marks]

## Markscheme

$$X \sim \text{Po}(m)$$

$$P(X = 2) = P(X < 2) \quad (M1)$$

$$\frac{1}{2}m^2e^{-m} = e^{-m}(1 + m) \quad (A1)(A1)$$

$$m = 2.73 \quad (1 + \sqrt{3}) \quad A1$$

in four hours the expected value is 10.9

$$(4 + 4\sqrt{3}) \quad A1$$

**Note:** Value of  $m$  does not need to be rounded.

[5 marks]

## Examiners report

Many candidates did not attempt this question and many others did not go beyond setting the equation up. Among the ones who attempted to solve the equation, once again, very few candidates took real advantage of GDC use to obtain the correct answer.

24a.

[6 marks]

## Markscheme

(i)

$$X \sim \text{Po}(11) \quad (M1)$$

$$P(X \leq 11) = 0.579 \quad (M1)A1$$

(ii)

$$P(X > 8 | x < 12) = \quad (M1)$$

$$= \frac{P(8 < X < 12)}{P(X < 12)} \quad \left( \text{or } \frac{P(X \leq 11) - P(X \leq 8)}{P(X \leq 11)} \text{ or } \frac{0.3472...}{0.5792...} \right) \quad A1$$

$$= 0.600 \quad A1 \quad N2$$

[6 marks]

## Examiners report

Generally, candidates had difficulties with this question, mainly in applying conditional probability and interpreting the expressions ‘more than’, ‘at least’ and ‘under’ to obtain correct expressions. Although many candidates identified the binomial distribution in part (b) (ii), very few succeeded in answering this question due to incorrect interpretation of the question or due to accuracy errors.

24b.

[10 marks]

## Markscheme

(i)

$$Y \sim \text{Po}(m)$$

$$P(Y > 3) = 0.24 \quad (M1)$$

$$P(Y \leq 3) = 0.76 \quad (M1)$$

$$e^{-m} \left(1 + m + \frac{1}{2}m^2 + \frac{1}{6}m^3\right) = 0.76 \quad (A1)$$

**Note:** At most two of the above lines can be implied.

Attempt to solve equation with GDC  $(M1)$

$$m = 2.49 \quad A1$$

(ii)

$$A \sim \text{Po}(4.98)$$

$$P(A > 5) = 1 - P(A \leq 5) = 0.380... \quad M1A1$$

$$W \sim B(4, 0.380...) \quad (M1)$$

$$P(W \geq 2) = 1 - P(W \leq 1) = 0.490 \quad M1A1$$

[10 marks]

## Examiners report

Generally, candidates had difficulties with this question, mainly in applying conditional probability and interpreting the expressions ‘more than’, ‘at least’ and ‘under’ to obtain correct expressions. Although many candidates identified the binomial distribution in part (b) (ii), very few succeeded in answering this question due to incorrect interpretation of the question or due to accuracy errors.

24c.

[6 marks]

## Markscheme

$$P(A < 25) = 0.8, P(A < 18) = 0.4$$

$$\frac{25-\mu}{\sigma} = 0.8416... \quad (M1)(A1)$$

$$\frac{18-\mu}{\sigma} = -0.2533... \quad (\text{or } -0.2534 \text{ from tables}) \quad (M1)(A1)$$

solving these equations  $(M1)$

$$\mu = 19.6 \quad A1$$

**Note:** Accept just 19.6, 19 or 20; award A0 to any other final answer.

[6 marks]

## Examiners report

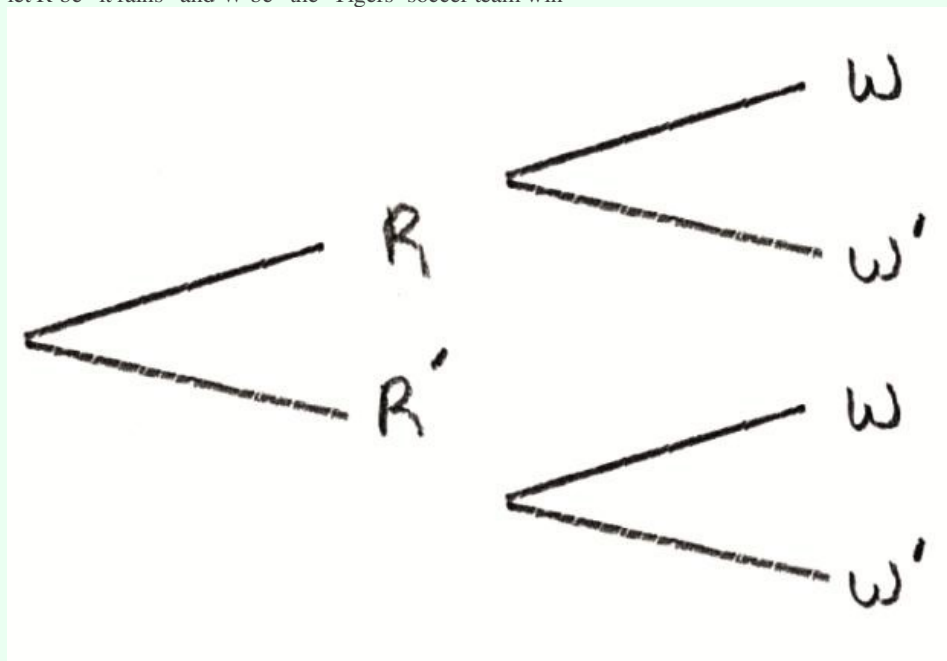
Generally, candidates had difficulties with this question, mainly in applying conditional probability and interpreting the expressions ‘more than’, ‘at least’ and ‘under’ to obtain correct expressions. Although many candidates identified the binomial distribution in part (b) (ii), very few succeeded in answering this question due to incorrect interpretation of the question or due to accuracy errors.



25a. [1 mark]

## Markscheme

let R be “it rains” and W be “the ‘Tigers’ soccer team win”



*AI*

[1 mark]

## Examiners report

This question was well answered in general.

25b. [2 marks]

## Markscheme

$$P(W) = \frac{2}{5} \times \frac{2}{7} + \frac{3}{5} \times \frac{4}{7} \quad (M1)$$

$$= \frac{16}{35} \quad AI$$

[2 marks]

## Examiners report

This question was well answered in general.

25c. [2 marks]

## Markscheme

$$P(R|W) = \frac{\frac{2}{5} \times \frac{2}{7}}{\frac{16}{35}} \quad (M1)$$

$$= \frac{1}{4} \quad AI$$

[2 marks]

## Examiners report

This question was well answered in general.

26a. [2 marks]

## Markscheme

$$f(x) \geq \frac{1}{25} \quad AI$$

$$g(x) \in \mathbb{R}, g(x) \geq 0 \quad AI$$

[2 marks]

## Examiners report

In (a), the ranges were often given incorrectly, particularly the range of  $g$  where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for  $f \circ g(x)$ . Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of  $X$ .

26b.

[4 marks]

## Markscheme

$$\begin{aligned} f \circ g(x) &= \frac{2\left(\frac{3x-4}{10}\right)^2 + 3}{75} & MIAI \\ &= \frac{\frac{2(9x^2 - 24x + 16)}{100} + 3}{75} & (AI) \\ &= \frac{9x^2 - 24x + 166}{3750} & AI \end{aligned}$$

[4 marks]

## Examiners report

In (a), the ranges were often given incorrectly, particularly the range of  $g$  where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for  $f \circ g(x)$ . Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of  $X$ .

26c.

[4 marks]

## Markscheme

(i) **METHOD 1**

$$\begin{aligned} y &= \frac{2x^2+3}{75} \\ x^2 &= \frac{75y-3}{2} & MI \\ x &= \sqrt{\frac{75y-3}{2}} & (AI) \end{aligned}$$

$$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}} \quad AI$$

**Note:** Accept  $\pm$  in line 3 for the **(AI)** but not in line 4 for the **AI**.

Award the **AI** only if written in the form

$$f^{-1}(x) = .$$

**METHOD 2**

$$\begin{aligned} y &= \frac{2x^2+3}{75} \\ x &= \frac{2y^2+3}{75} & MI \\ y &= \sqrt{\frac{75x-3}{2}} & (AI) \end{aligned}$$

$$\Rightarrow f^{-1}(x) = \sqrt{\frac{75x-3}{2}} \quad AI$$

**Note:** Accept  $\pm$  in line 3 for the **(AI)** but not in line 4 for the **AI**.

Award the **AI** only if written in the form

$$f^{-1}(x) = .$$

(ii) domain:

$$x \geq \frac{1}{25}; \text{ range:}$$

$$f^{-1}(x) \geq 0 \quad AI$$

[4 marks]

## Examiners report

In (a), the ranges were often given incorrectly, particularly the range of  $g$  where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for  $f \circ g(x)$ . Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of  $X$ .

26d. [6 marks]

## Markscheme

probabilities from

$f(x)$  :

$X$	0	1	2	3	4
$P(X=x)$	$\frac{3}{75}$	$\frac{5}{75}$	$\frac{11}{75}$	$\frac{21}{75}$	$\frac{35}{75}$

A2

**Note:** Award **A1** for one error, **A0** otherwise.

probabilities from

$g(x)$  :

$X$	0	1	2	3	4
$P(X=x)$	$\frac{4}{10}$	$\frac{1}{10}$	$\frac{2}{10}$	$\frac{5}{10}$	$\frac{8}{10}$

A2

**Note:** Award **A1** for one error, **A0** otherwise.

only in the case of

$f(x)$  does

$\sum P(X=x) = 1$ , hence only

$f(x)$  can be used as a probability mass function A2

[6 marks]

## Examiners report

In (a), the ranges were often given incorrectly, particularly the range of  $g$  where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for  $f \circ g(x)$ . Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of  $X$ .

26e. [2 marks]

## Markscheme

$$E(x) = \sum x \cdot P(X=x) \quad \text{M1}$$

$$= \frac{5}{75} + \frac{22}{75} + \frac{63}{75} + \frac{140}{75} = \frac{230}{75} \left( = \frac{46}{15} \right) \quad \text{A1}$$

[2 marks]

## Examiners report

In (a), the ranges were often given incorrectly, particularly the range of  $g$  where the modulus signs appeared to cause difficulty. In (b), it was disappointing to see so many candidates making algebraic errors in attempting to determine the expression for  $f \circ g(x)$ . Many candidates were unable to solve (d) correctly with arithmetic errors and incorrect reasoning often seen. Since the solution to (e) depended upon a correct choice of function in (d), few correct solutions were seen with some candidates even attempting to use integration, inappropriately, to find the mean of  $X$ .

27a. [1 mark]

## Markscheme

$$E(X) = np$$

$$\Rightarrow 10 = 30p$$

$$\Rightarrow p = \frac{1}{3} \quad \text{AI}$$

[1 mark]

## Examiners report

Again this proved to be an accessible question for students with many students gaining full marks. Most candidates used the calculator to find the answers to parts (b) and (c) which is what was intended, but candidates should be aware that there are often marks for recognising what needs to be found, even if the candidate does not obtain the final correct answer. It is suggested that in this style of question, candidates should indicate what they are trying to find as well as giving the final answer.

27b. [2 marks]

## Markscheme

$$P(X = 10) = \binom{30}{10} \left(\frac{1}{3}\right)^{10} \left(\frac{2}{3}\right)^{20} = 0.153 \quad (M1)AI$$

[2 marks]

## Examiners report

Again this proved to be an accessible question for students with many students gaining full marks. Most candidates used the calculator to find the answers to parts (b) and (c) which is what was intended, but candidates should be aware that there are often marks for recognising what needs to be found, even if the candidate does not obtain the final correct answer. It is suggested that in this style of question, candidates should indicate what they are trying to find as well as giving the final answer.

27c. [2 marks]

## Markscheme

$$P(X \geq 15) = 1 - P(X \leq 14) \quad (M1)$$

$$= 1 - 0.9565... = 0.0435 \quad AI$$

[2 marks]

## Examiners report

Again this proved to be an accessible question for students with many students gaining full marks. Most candidates used the calculator to find the answers to parts (b) and (c) which is what was intended, but candidates should be aware that there are often marks for recognising what needs to be found, even if the candidate does not obtain the final correct answer. It is suggested that in this style of question, candidates should indicate what they are trying to find as well as giving the final answer.

28. [6 marks]

## Markscheme

$$\frac{\sum_{i=1}^{15} x_i}{15} = 11.5 \Rightarrow \sum_{i=1}^{15} x_i = 172.5 \quad (AI)$$

new mean

$$= \frac{172.5 - 22.1}{14} \quad (M1)$$

$$= 10.7428... = 10.7 \text{ (3sf)} \quad AI$$

$$\frac{\sum_{i=1}^{15} x_i^2}{15} - 11.5^2 = 9.3 \quad (M1)$$

$$\Rightarrow \sum_{i=1}^{15} x_i^2 = 2123.25$$

new variance

$$= \frac{2123.25 - 22.1^2}{14} - (10.7428...)^2 \quad (M1)$$

$$= 1.37 \text{ (3sf)} \quad AI$$

[6 marks]

## Examiners report

Most candidates were successful in finding the correct value of the mean; however, the variance caused many difficulties. Many candidates affirmed that there were no differences in the variance as it remained constant; some others got wrong results due to premature rounding of figures. Many candidates lost the final mark because they rounded their answers prematurely, resulting in a very inaccurate answer to this question.

29a. [2 marks]

## Markscheme

$$P(WWW) = 0.75 \times 0.375 \times 0.1875 = 0.0527 \text{ (3sf)} \quad \left(\frac{3}{4} \times \frac{3}{8} \times \frac{3}{16} = \frac{27}{512}\right) \quad (M1)A1$$

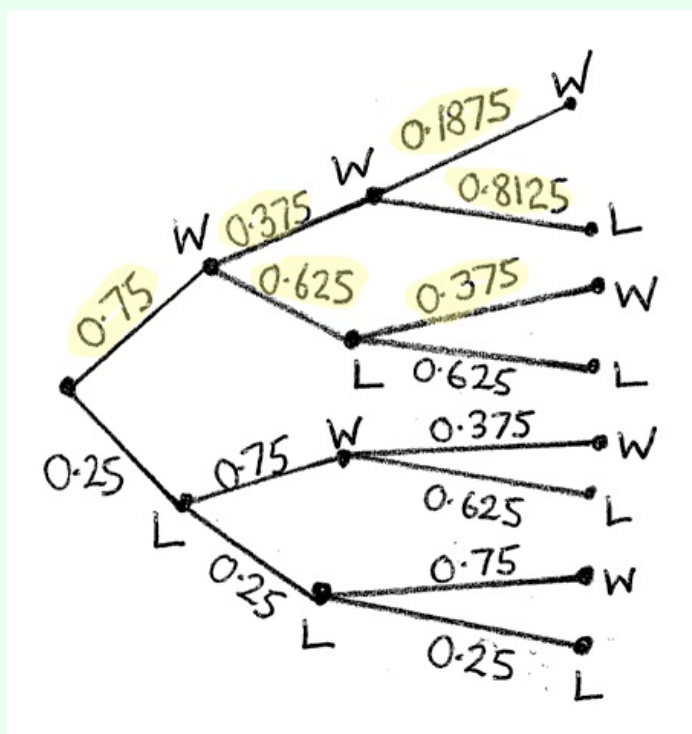
[2 marks]

## Examiners report

Part (a) was generally successful to most candidates; however the conditional probability was proved difficult to many candidates either because the unconditional probability of two correct games was found or the success in the second and third game was included. Many candidates used a clear tree diagram to calculate the corresponding probabilities. However other candidates frequently tried to do the problem without drawing a tree diagram and often had incorrect probabilities. It was sad to read many answers with probabilities greater than 1.

29b. [6 marks]

## Markscheme



(M1)(A1)

**Note:** Award **M1** for any reasonable attempt to use a tree diagram showing that three games were played (do not award **M1** for tree diagrams that only show the first two games) and **A1** for the highlighted probabilities.

$$\begin{aligned} P(\text{wins 2 games} | \text{wins first game}) &= \frac{P(WWL, WLW)}{P(\text{wins first game})} \quad (M1) \\ &= \frac{0.75 \times 0.375 \times 0.8125 + 0.75 \times 0.625 \times 0.375}{0.75} \quad (A1)(A1) \\ &= 0.539 \text{ (3sf)} \quad \left(\text{or } \frac{69}{128}\right) \quad A1 \end{aligned}$$

**Note:** Candidates may use the tree diagram to obtain the answer without using the conditional probability formula, ie,

$$P(\text{wins 2 games} | \text{wins first game}) = 0.375 \times 0.8125 + 0.625 \times 0.375 = 0.539.$$

[6 marks]

## Examiners report

Part (a) was generally successful to most candidates; however the conditional probability was proved difficult to many candidates either because the unconditional probability of two correct games was found or the success in the second and third game was included. Many candidates used a clear tree diagram to calculate the corresponding probabilities. However other candidates frequently tried to do the problem without drawing a tree diagram and often had incorrect probabilities. It was sad to read many answers with probabilities greater than 1.

30a. [2 marks]

## Markscheme

$$2.2 \times 6 \times 60 = 792 \quad (M1)A1$$

[2 marks]

## Examiners report

This question was generally well done by most candidates. It was evident that candidates had been well prepared in Poisson and normal distribution. In parts (a)-(d) candidates were usually successful and appropriate methods were shown although many candidates used labored algebraic approaches to solving simultaneous equations and wasted time answering part (d). Part (e) was very well answered by a smaller number of candidates but it was obviously more demanding in its level of abstraction.

30b. [3 marks]

## Markscheme

$$V \sim \text{Po}(2.2 \times 60) \quad (M1)$$

$$P(V > 100) = 0.998 \quad (M1)A1$$

[3 marks]

## Examiners report

This question was generally well done by most candidates. It was evident that candidates had been well prepared in Poisson and normal distribution. In parts (a)-(d) candidates were usually successful and appropriate methods were shown although many candidates used labored algebraic approaches to solving simultaneous equations and wasted time answering part (d). Part (e) was very well answered by a smaller number of candidates but it was obviously more demanding in its level of abstraction.

30c. [2 marks]

## Markscheme

$$(0.997801\dots)^6 = 0.987 \quad (M1)A1$$

[2 marks]

## Examiners report

This question was generally well done by most candidates. It was evident that candidates had been well prepared in Poisson and normal distribution. In parts (a)-(d) candidates were usually successful and appropriate methods were shown although many candidates used labored algebraic approaches to solving simultaneous equations and wasted time answering part (d). Part (e) was very well answered by a smaller number of candidates but it was obviously more demanding in its level of abstraction.

30d. [6 marks]

## Markscheme

$A \sim N(\mu, \sigma^2)$   
 $P(A < 35) = 0.29$  and  $P(A > 55) = 0.23 \Rightarrow P(A < 55) = 0.77$   
 $P\left(Z < \frac{35-\mu}{\sigma}\right) = 0.29$  and  $P\left(Z < \frac{55-\mu}{\sigma}\right) = 0.77$  (M1)  
 use of inverse normal (M1)  
 $\frac{35-\mu}{\sigma} = -0.55338\dots$  and  $\frac{55-\mu}{\sigma} = 0.738846\dots$  (A1)  
 solving simultaneously (M1)  
 $\mu = 43.564\dots$  and  $\sigma = 15.477\dots$  A1A1  
 $\mu = 43.6$  and  $\sigma = 15.5$  (3sf)  
 [6 marks]

## Examiners report

This question was generally well done by most candidates. It was evident that candidates had been well prepared in Poisson and normal distribution. In parts (a)-(d) candidates were usually successful and appropriate methods were shown although many candidates used labored algebraic approaches to solving simultaneous equations and wasted time answering part (d). Part (e) was very well answered by a smaller number of candidates but it was obviously more demanding in its level of abstraction.

30e. [5 marks]

## Markscheme

$0.29n = 100 \Rightarrow n = 344.82\dots$  (M1)(A1)  
 $P(A < 50) = 0.66121\dots$  (A1)  
 expected number of visitors under 50 = 228 (M1)A1  
 [5 marks]

## Examiners report

This question was generally well done by most candidates. It was evident that candidates had been well prepared in Poisson and normal distribution. In parts (a)-(d) candidates were usually successful and appropriate methods were shown although many candidates used labored algebraic approaches to solving simultaneous equations and wasted time answering part (d). Part (e) was very well answered by a smaller number of candidates but it was obviously more demanding in its level of abstraction.

31a. [1 mark]

## Markscheme

$B$  has the binomial distribution  
 $\left(B\left(5, \frac{4}{10}\right)\right)$  A1  
 [1 mark]

## Examiners report

This was generally well answered. Some students did not read the question carefully enough and see the comparisons made between the Hypergeometric distribution and the Binomial distribution, with 5 trials (some candidates went to 10 trials) in each case. Part (h) caused the most problems and it was very rare to see a script that gained the reasoning mark for saying that  $A$  and  $B$  were independent events. This question was a good indicator of the standard of the rest of the paper.

31b. [2 marks]

## Markscheme

$$P(B=3) = \left( \binom{5}{3} \left( \frac{4}{10} \right)^3 \left( \frac{6}{10} \right)^2 \right) \frac{144}{625} (= 0.2304) \quad (M1)A1$$

**Note:** Accept 0.230.

[2 marks]

## Examiners report

This was generally well answered. Some students did not read the question carefully enough and see the comparisons made between the Hypergeometric distribution and the Binomial distribution, with 5 trials (some candidates went to 10 trials) in each case. Part (h) caused the most problems and it was very rare to see a script that gained the reasoning mark for saying that  $A$  and  $B$  were independent events. This question was a good indicator of the standard of the rest of the paper.

31c. [2 marks]

## Markscheme

$$P(B=5) = \left( \left( \frac{4}{10} \right)^5 \right) \frac{32}{3125} (= 0.01024) \quad (M1)A1$$

**Note:** Accept 0.0102.

[2 marks]

## Examiners report

This was generally well answered. Some students did not read the question carefully enough and see the comparisons made between the Hypergeometric distribution and the Binomial distribution, with 5 trials (some candidates went to 10 trials) in each case. Part (h) caused the most problems and it was very rare to see a script that gained the reasoning mark for saying that  $A$  and  $B$  were independent events. This question was a good indicator of the standard of the rest of the paper.

32. [6 marks]

## Markscheme

(a)

$$X \sim B(n, 0.4) \quad (A1)$$

Using

$$P(X=x) = \binom{n}{r} (0.4)^x (0.6)^{n-x} \quad (M1)$$

$$P(X=2) = \binom{n}{2} (0.4)^2 (0.6)^{n-2}$$

$$\left( = \frac{n(n-1)}{2} (0.4)^2 (0.6)^{n-2} \right) \quad A1 \quad N3$$

$$(b) \quad P(X=2) = 0.121 \quad A1$$

Using an appropriate method (including trial and error) to solve their equation.  $(M1)$

$$n = 10 \quad A1 \quad N2$$

**Note:** Do not award the last  $A1$  if any other solution is given in their final answer.

[6 marks]



## Examiners report

Part (a) was generally well done. The most common error was to omit the binomial coefficient *i.e.* not identifying that the situation is described by a binomial distribution.

Finding the correct value of  $n$  in part (b) proved to be more elusive. A significant proportion of candidates attempted algebraic approaches and seemingly did not realise that the equation could only be solved numerically. Candidates who obtained  $n = 10$  often accomplished this by firstly attempting to solve the equation algebraically before ‘resorting’ to a GDC approach. Some candidates did not specify their final answer as an integer while others stated  $n = 1.76$  as their final answer.

33a. [7 marks]

## Markscheme

(i)

$$P(4.8 < X < 7.5) = P(-0.8 < Z < 1) \quad (M1)$$

$$= 0.629 \quad A1 \quad N2$$

**Note:** Accept

$$P(4.8 \leq X \leq 7.5) = P(-0.8 \leq Z \leq 1) .$$

(ii) Stating

$$P(X < d) = 0.15 \text{ or sketching an appropriately labelled diagram.} \quad A1$$

$$\frac{d-6}{1.5} = -1.0364 \dots \quad (M1)(A1)$$

$$d = (-1.0364 \dots)(1.5) + 6 \quad (M1)$$

$$= 4.45 \text{ (km)} \quad A1 \quad N4$$

[7 marks]

## Examiners report

This question was generally well done despite a large proportion of candidates being awarded an accuracy penalty. Candidates found part (a) (i) to be quite straightforward and was generally done very well. In part (a) (ii), a number of candidates used

$$\frac{d-6}{1.5} = 1.0364 \dots \text{ instead of}$$

$\frac{d-6}{1.5} = -1.0364 \dots$  In part (b), a pleasingly high number of candidates were able to set up and solve a pair of simultaneous linear equations to correctly find the values of

$\mu$  and

$\sigma$ . Some candidates prematurely rounded intermediate results. In part (c), a number of candidates were unable to express a correct Poisson inequality. Common errors included stating

$$P(T \geq 3) = 1 - P(T \leq 3) \text{ and using}$$

$$\mu = 7.$$

## Markscheme

Stating **both**

$$P(X > 8) = 0.1 \text{ and}$$

$$P(X < 2) = 0.05 \text{ or sketching an appropriately labelled diagram. } \textbf{R1}$$

Setting up two equations in

$\mu$  and

$\sigma$  (M1)

$$8 =$$

$$\mu + (1.281\dots)$$

$$\sigma \text{ and } 2 =$$

$$\mu - (1.644\dots)$$

$\sigma$  A1

Attempting to solve for

$\mu$  and

$\sigma$  (including by graphical means) (M1)

$$\sigma = 2.05 \text{ (km) and}$$

$$\mu = 5.37 \text{ (km) A1A1 N4}$$

**Note:** Accept

$$\mu = 5.36, 5.38 \dots$$

[6 marks]

## Examiners report

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$\mu$  and

$\sigma$ . Some candidates prematurely rounded intermediate results. In part (c), a number of candidates were unable to express a correct Poisson inequality. Common errors included stating

$$P(T \geq 3) = 1 - P(T \leq 3) \text{ and using}$$

$$\mu = 7.$$

## Markscheme

(i) Use of the Poisson distribution in an inequality. *MI*

$$P(T \geqslant 3) = 1 - P(T \leqslant 2) \quad (AI)$$

$$= 0.679\dots \quad AI$$

Required probability is

$$(0.679\dots)^2 = 0.461 \quad MIAI \quad N3$$

**Note:** Allow *FT* for their value of

$$P(T \geqslant 3) \, .$$

(ii)

$$\tau \sim \text{Po}(17.5) \quad AI$$

$$P(\tau = 15) = \frac{\text{e}^{-17.5}(17.5)^{15}}{15!} \quad (MI)$$

$$= 0.0849 \quad AI \quad N2$$

[8 marks]

## Examiners report

This question was generally well done despite a large proportion of candidates being awarded an accuracy penalty. Candidates found part (a) (i) to be quite straightforward and was generally done very well. In part (a) (ii), a number of candidates used

$$\frac{d-6}{1.5} = 1.0364\dots \text{ instead of}$$

$\frac{d-6}{1.5} = -1.0364\dots$  In part (b), a pleasingly high number of candidates were able to set up and solve a pair of simultaneous linear equations to correctly find the values of

$\mu$  and

$\sigma$ . Some candidates prematurely rounded intermediate results. In part (c), a number of candidates were unable to express a correct Poisson inequality. Common errors included stating

$$P(T \geqslant 3) = 1 - P(T \leqslant 3) \text{ and using}$$

$$\mu = 7.$$

## Markscheme

$$P(\text{no heads from } n \text{ coins tossed}) =$$

$$0.5^n \quad (AI)$$

$$P(\text{no head}) =$$

$$\frac{1}{3} \times \frac{1}{2} + \frac{1}{3} \times \frac{1}{4} + \frac{1}{3} \times \frac{1}{8} \quad MI$$

$$=$$

$$\frac{7}{24} \quad AI$$

[3 marks]

## Examiners report

[N/A]

34b. [3 marks]

## Markscheme

$$\begin{aligned} P(2 \mid \text{no heads}) &= \frac{P(2 \text{ coins and no heads})}{P(\text{no heads})} \quad \textbf{M1} \\ &= \frac{\frac{1}{12}}{\frac{7}{24}} \quad \textbf{A1} \\ &= \frac{2}{7} \quad \textbf{A1} \end{aligned}$$

[3 marks]

## Examiners report

[N/A]

35a. [3 marks]

## Markscheme

$$\begin{aligned} E(X) &= \int_0^1 12x^3(1-x)dx \quad \textbf{M1} \\ &= 12 \left[ \frac{x^4}{4} - \frac{x^5}{5} \right]_0^1 \quad \textbf{A1} \\ &= \frac{3}{5} \quad \textbf{A1} \end{aligned}$$

[3 marks]

## Examiners report

[N/A]

35b. [3 marks]

## Markscheme

$$f'(x) = 12(2x - 3x^2) \quad \textbf{A1}$$

at the mode

$$f'(x) = 12(2x - 3x^2) = 0 \quad \textbf{M1}$$

therefore the mode

$$= \frac{2}{3} \quad \textbf{A1}$$

[3 marks]

## Examiners report

[N/A]

36a. [4 marks]

## Markscheme

we are given that

$$2.1 = \mu - 0.5244\sigma$$

$$2.5 = \mu + 0.6745\sigma \quad \text{MIAI}$$

$$\mu = 2.27, \sigma = 0.334 \quad \text{AIAI}$$

[4 marks]

## Examiners report

[N/A]

36b. [5 marks]

## Markscheme

(i) let  $X$  denote the number of birds weighing more than 2.5 kg

then  $X$  is  $B(10, 0.25)$  *AI*

$$E(X) = 2.5 \quad \text{AI}$$

(ii) 0.0584 *AI*

(iii) to find the most likely value of  $X$ , consider

$$p_0 = 0.0563\dots, p_1 = 0.1877\dots, p_2 = 0.2815\dots, p_3 = 0.2502\dots \quad \text{MI}$$

therefore, most likely value = 2 *AI*

[5 marks]

## Examiners report

[N/A]

36c. [8 marks]

### Markscheme

(i) we solve

$$1 - P(Y \leq 1) = 0.80085 \text{ using the GDC} \quad \text{M1}$$

$$\lambda = 3.00 \quad \text{A1}$$

(ii) let

$X_1, X_2$  denote the number of eggs laid by each bird

$$P(X_1 + X_2 = 2) = P(X_1 = 0)P(X_2 = 1) + P(X_1 = 1)P(X_2 = 1) + P(X_1 = 2)P(X_2 = 0) \quad \text{M1A1}$$

$$= e^{-3} \times e^{-3} \times \frac{9}{2} + (e^{-3} \times 3)^2 + e^{-3} \times \frac{9}{2} \times e^{-3} = 0.0446 \quad \text{A1}$$

(iii)

$$P(X_1 = 1, X_2 = 1 | X_1 + X_2 = 2) = \frac{P(X_1=1, X_2=1)}{P(X_1+X_2=2)} \quad \text{M1A1}$$

$$= 0.5 \quad \text{A1}$$

[8 marks]

### Examiners report

[N/A]

37a. [2 marks]

### Markscheme

$$P(X = 1) = \frac{1}{3} \quad \text{A1}$$

$$P(X = 2) = \frac{2}{3} \times \frac{1}{4} \quad \text{A1}$$

$$= \frac{1}{6} \quad \text{AG}$$

[2 marks]

### Examiners report

[N/A]

37b. [6 marks]

### Markscheme

$$G(t) = \frac{1}{3}t + \frac{2}{3} \times \frac{1}{4}t^2 + \frac{2}{3} \times \frac{3}{4} \times \frac{1}{3}t^3 + \frac{2}{3} \times \frac{3}{4} \times \frac{2}{3} \times \frac{1}{4}t^4 + \dots \quad \text{M1A1}$$

$$= \frac{1}{3}t \left(1 + \frac{1}{2}t^2 + \dots\right) + \frac{1}{6}t^2 \left(1 + \frac{1}{2}t^2 + \dots\right) \quad \text{M1A1}$$

$$= \frac{\frac{t}{3}}{1 - \frac{t^2}{2}} + \frac{\frac{t^2}{6}}{1 - \frac{t^2}{2}} \quad \text{A1A1}$$

$$= \frac{2t + t^2}{6 - 3t^2} \quad \text{AG}$$

[6 marks]

### Examiners report

[N/A]

37c. [4 marks]

## Markscheme

$$G'(t) = \frac{(2+2t)(6-3t^2)+6t(2t+t^2)}{(6-3t^2)^2} \quad MIAI$$

$$E(X) = G'(1) = \frac{10}{3} \quad MIAI$$

[4 marks]

## Examiners report

[N/A]

38a. [4 marks]

## Markscheme

$$E(X) = 1 \times \theta + 2 \times 2\theta + 3(1 - 3\theta) = 3 - 4\theta \quad MIAI$$

$$\text{Var}(X) = 1 \times \theta + 4 \times 2\theta + 9(1 - 3\theta) - (3 - 4\theta)^2 \quad MIAI$$

$$= 6\theta - 16\theta^2 \quad AG$$

[4 marks]

## Examiners report

[N/A]

38b. [10 marks]

## Markscheme

(i)

$$E(\hat{\theta}_1) = \frac{3 - E(\bar{X})}{4} = \frac{3 - (3 - 4\theta)}{4} = \theta \quad MIAI$$

so

$\hat{\theta}_1$  is an unbiased estimator of

$\theta \quad AG$

$$\text{Var}(\hat{\theta}_1) = \frac{6\theta - 16\theta^2}{16n} \quad AI$$

(ii) each of the  $n$  observed values has a probability

$\theta$  of having the value 1  $RI$

so

$$Y \sim B(n, \theta) \quad AG$$

$$E(\hat{\theta}_2) = \frac{E(Y)}{n} = \frac{n\theta}{n} = \theta \quad AI$$

$$\text{Var}(\hat{\theta}_2) = \frac{n\theta(1-\theta)}{n^2} = \frac{\theta(1-\theta)}{n} \quad MIAI$$

(iii)

$$\text{Var}(\hat{\theta}_1) - \text{Var}(\hat{\theta}_2) = \frac{6\theta - 16\theta^2 - 16\theta + 16\theta^2}{16n} \quad MI$$

$$= \frac{-10\theta}{16n} < 0 \quad AI$$

$\hat{\theta}_1$  is the more efficient estimator since it has the smaller variance  $RI$

[10 marks]

## Examiners report

[N/A]

